Fusion of Threshold Rules for Target Detection in Wireless Sensor Networks

MENGXIA ZHU
Southern Illinois University
SONG DING
Louisiana State University
QISHI WU
University of Memphis
R. R. BROOKS
Clemson University
N. S. V. RAO
Oak Ridge National Laboratory
and
S. S. IYENGAR
Louisiana State University

We propose a binary decision fusion rule that reaches a global decision on the presence of a target by integrating local decisions made by multiple sensors. Without requiring a priori probability of target presence, the fusion threshold bounds derived using Chebyshev's inequality ensure a higher hit rate and lower false alarm rate compared to the weighted averages of individual sensors. The Monte Carlo-based simulation results show that the proposed approach significantly improves target detection performance, and can also be used to guide the

R. R. Brooks acknowledges financial support by ARO through grant W911 NF-05-1-0226.
Authors' addresses: M. Zhu, Department of Computer Science, Southern Illinois University, Carbondale, IL 62901; email: mzhu@cs.siu.edu (corresponding author). S. Ding, S. S. Iyengar, Department of Computer Science, Louisiana State University, Baton Rouge, LA 70803; email: {iyengar,sding}@bit.csc.lsu.edu. Q. Wu, Department of Computer Science, University of Memphis, Memphis, TN 38152; email: qishiwu@memphis.edu, R. R. Brooks, Department of Computer Engineering, Clemson University, Clemson, SC 61801; email: rrb@acm.org. N. S. V. Rao, Computer Science and Mathematics Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831; email: raons@ornl.gov.

DOI 10.1145/1689239.1689248 http://doi.acm.org/10.1145/1689239.1689248

actual threshold selection in practical sensor network implementation under certain error rate constraints.

Categories and Subject Descriptors: I.2.9 [**Artificial Intelligence**]: Robotics–Sensors

General Terms: Algorithms, Performance

Additional Key Words and Phrases: Hit rate, false alarm rate, Chebyshev inequality, ROC curve, wireless sensor network, binary decision fusion

ACM Reference Format:

Zhu, M., Ding, S., Wu, Q., Brooks, R. R., Rao, N. S. V., and Iyengar, S. S. 2010. Fusion of threshold rules for target detection in wireless sensor networks. ACM Trans. Sensor Netw. 6, 2, Article 18 (February 2010), 7 pages.
DOI = 10.1145/1689239.1689248 http://doi.acm.org/10.1145/1689239.1689248

1. INTRODUCTION

Many existing non-model-based or model-based fusion methodologies are derived from some variants of decision rules such as Voting, Bayes Criterion, Maximum a Posterior Criterion (MAP), and Neyman-Pearson [Tenney and Sandell 1981; Sadjadi 1986; Thomopoulos et al. 1987; Reibman and Nolte 1987a, 1987b; Varshney 1997; Tsitsiklis 1993; Niu et al. 2006; Chen and Varshney 2007; Katenka et al. 2008; Duarte and Hu 2004]. Data fusion is in general categorized as low-, intermediate-, or high- level fusion, depending on the stage where actual fusion processing takes place. In this technical note, we present a model based high-level hard fusion scheme, also known as decision fusion, where a final global decision is reached by integrating local binary decisions made by multiple sensor nodes that detect the same target from different distances. Without requiring a priori knowledge on the probability of target presence, this centralized fusion scheme uses Chebyshev's inequality to derive threshold bounds that ensure a better system performance compared with the weighted averages of all individual sensors. Simulation results based on Monte Carlo method show that the error probabilities in the fused system are significantly reduced to near zero. Furthermore, the simulation results are particularly useful in guiding practical implementation in which an upper bound on the false alarm rate and minimization of missing rate or vice versa are desired at the same time. The rest of the article is organized as follows: Mathematical model is presented in Section 2. Section 3 discusses the technical approach to derive the proper system threshold bounds. Simulation results are given in Section 4. We conclude our work in Section 5.

2. PROBLEM FORMULATION

We consider N sensor nodes randomly deployed in a region of interest (ROI) with radius R. Noise at each local sensor follows the standard Gaussian distribution defined as: $n_i \sim \aleph(0,1)$. Each sensor makes a binary local decision choosing between:

$$H_1: \quad r_i = s_i + n_i \quad H_0: \quad r_i = n_i, \tag{1}$$

where H_1 is the hypothesis of target presence and H_0 is the hypothesis of target absence. r_i is the total sensor reading of sensor i and n_i denotes the noise level observed by sensor i. The signal strength s_i decays as the sensor moves away from the target and follows the isotropic attenuation power model as defined in Equation 2:

$$s_i = \frac{S_0}{\sqrt{1+\beta d_i^m}}, \tag{2}$$

where S_0 is the original signal power from the target, β is a constant, and d_i represents the Euclidean distance between the target and sensor i. The signal attenuation exponent m ranges from 2 to 3. Here we assume the same threshold τ for every sensor node because some simple sensor motes may not have the intelligence and necessary processing resources to adjust their thresholds dynamically. Thus, the hit rate p_{h_i} and false alarm rate p_{f_i} for sensor i can be defined as:

$$p_{h_i} = \int_{\tau}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-(x-s_i)^2}{2}} dx, \quad p_{f_i} = \int_{\tau}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx. \tag{3}$$

Our fusion method can be applied to any signal attenuation model. A simplified model is used in Equation 2 and Equation 3 simply for discussion and simulation purposes. We assume that factory manufactured sensors are calibrated with specified ROC curves fitted for the characteristics of desired target under certain environment in real applications.

3. THRESHOLD FUSION METHOD

Sensor i makes an independent binary decision S_i as either 0 or 1. The fusion center uses a simple 0/1 counting rule for convenience and collects local decisions and computes S as: $S = \sum_{i=1}^{N} S_i$, which is then compared with a system threshold T to make a final decision.[1] For simplicity, we neglect covariance and assume that sensor measurements are conditionally independent under H_1. The mean and variance of S are given below when the target is present:

$$E(S|H_1) = \sum_{i=1}^{N} p_{h_i}, \quad Var(S|H_1) = \sum_{i=1}^{N} p_{h_i}(1-p_{h_i}). \tag{4}$$

Similarly, the mean and variance of S when a target is absent are defined as:

$$E(S|H_0) = \sum_{i=1}^{N} p_{f_i}, \quad Var(S|H_0) = \sum_{i=1}^{N} p_{f_i}(1-p_{f_i}). \tag{5}$$

The threshold value T is critical to the system performance. Let P_h and P_f denote the hit rate and false alarm rate of the fused system respectively in Equation 6. We also make reasonable value bounds for T as $\sum_{\mathrm{i}=1}^{N} p_{f_i} < T < \sum_{\mathrm{i}=1}^{N} p_{h_i}$.

$$P_h = P\{S \geq \mathrm{T}|\mathrm{H}_1\}, \quad P_f = P\{S \geq \mathrm{T}|\mathrm{H}_0\} = 1 - P\{S < \mathrm{T}|\mathrm{H}_0\}, \tag{6}$$

[1]A simple suboptimal decision metric is used here without using weights to differentiate individual decisions S_i.

The weighted averages of p_{h_i} and p_{f_i}, $i = 1, 2, \ldots, N$ are defined as follows, respectively:

$$\sum_{i=1}^{N} \frac{p_{h_i}}{\sum_{j=1}^{N} p_{h_j}} p_{h_i} = \frac{\sum_{i=1}^{N} p_{h_i}^2}{\sum_{i=1}^{N} p_{h_i}}, \quad \sum_{i=1}^{N} \frac{1-p_{f_i}}{\sum_{j=1}^{N}(1-p_{f_j})} p_{f_i} = \frac{\sum_{i=1}^{N}(1-p_{f_i})p_{f_i}}{\sum_{i=1}^{N}(1-p_{f_i})}. \quad (7)$$

We wish to achieve better system detection performance than the corresponding weighted averages in terms of higher hit rate and lower false alarm rate. Thus, the following inequalities should hold:

$$P_h > \frac{\sum_{i=1}^{N} p_{h_i}^2}{\sum_{i=1}^{N} p_{h_i}}, \quad P_f < \frac{\sum_{i=1}^{N}(1-p_{f_i})p_{f_i}}{\sum_{i=1}^{N}(1-p_{f_i})}. \quad (8)$$

We first consider a lower bound on the hit rate of the fused system:

$$P_h \geq P\left\{\left|S - \sum_{\mathrm{i}=1}^{N} p_{h_i}\right| \leq \left(\sum_{\mathrm{i}=1}^{N} p_{h_i} - \mathrm{T}\right)\middle|\mathrm{H}_1\right\} \geq 1 - \frac{\sigma^2}{\mathrm{k}^2} = 1 - \frac{\sum_{i=1}^{N} p_{h_i}(1-p_{h_i})}{\left(\sum_{i=1}^{N} p_{h_i} - T\right)^2} \quad (9)$$

where we apply Chebyshev's inequality in the second step and denote $(\sum_1^{\mathrm{N}} p_{h_i} - \mathrm{T})$ by k. Now the inequality of P_h in Equation 8 can be ensured by the following sufficient condition:

$$1 - \frac{\sum_{i=1}^{N} p_{h_i}(1-p_{h_i})}{\left(\sum_{i=1}^{N} p_{h_i} - T\right)^2} \geq \frac{\sum_{i=1}^{N} p_{h_i}^2}{\sum_{i=1}^{N} p_{h_i}}. \quad (10)$$

Following that, an upper bound on T can be derived from Equation 10 as follows:

$$T \leq \sum_{i=1}^{N} p_{h_i} - \sqrt{\sum_{i=1}^{N} p_{h_i}}. \quad (11)$$

For the false alarm rate, we follow a similar procedure from Equation 12 to Equation 15 to compute the lower bound. Chebyshev's inequality is applied in the second step in Equation 13.

$$P\{S < T|\mathrm{H}_0\} \geq P\left\{\left|S - \sum_{i=1}^{N} p_{f_i}\right| \leq \left(T - \sum_{i=1}^{N} p_{f_i}\right)|\mathrm{H}_0\right\}, \quad (12)$$

$$P_f \leq 1 - P\left\{\left|S - \sum_{i=1}^{N} p_{f_i}\right| \leq \left(T - \sum_{i=1}^{N} p_{f_i}\right)|\mathrm{H}_0\right\} \leq \frac{\sum_{i=1}^{N} p_{f_i}(1-p_{f_i})}{\left(T - \sum_{i=1}^{N} p_{f_i}\right)^2}. \quad (13)$$

Now we consider the sufficient condition that ensures the system false alarm rate to be smaller than that of weighted average:

$$\frac{\sum_{i=1}^{N} p_{f_i}(1-p_{f_i})}{\left(T-\sum_{i=1}^{N} p_{f_i}\right)^2} \leq \frac{\sum_{i=1}^{N}(1-p_{f_i})p_{f_i}}{\sum_{i=1}^{N}(1-p_{f_i})}, \tag{14}$$

$$T \geq \sum_{i=1}^{N} p_{f_i} + \sqrt{\sum_{i=1}^{N}(1-p_{f_i})}. \tag{15}$$

We define the range of T using the upper bound in Equation 11 and lower bound in Equation 15:

$$\left[\sum_{i=1}^{N} p_{f_i} + \sqrt{\sum_{i=1}^{N}(1-p_{f_i})}, \sum_{i=1}^{N} p_{h_i} - \sqrt{\sum_{i=1}^{N} p_{h_i}}\right]. \tag{16}$$

To ensure that the upper bound is larger than the lower bound, we have the following restriction on individual hit rates, false alarm rates, and the number of sensor nodes:

$$\sum_{i=1}^{N} p_{f_i} + \sqrt{\sum_{i=1}^{N}(1-p_{f_i})} - \sum_{i=1}^{N} p_{h_i} + \sqrt{\sum_{i=1}^{N} p_{h_i}} \leq 0. \tag{17}$$

4. SIMULATION RESULTS

For performance evaluation, we used the Monte Carlo simulation method to produce the system's receiver operative characteristic (ROC) curve. The simulations consider 25 sensor nodes randomly deployed around a fixed target in ROI with a deployment radius ranging from 1 to 5 for comparing different deployment strategies. We assume that all sensors have a uniform signal threshold for making a local binary decision. The sensor placed right next to the target is assumed to have the highest hit rate of 0.75 and the corresponding false alarm rate is 0.2. Sensors deployed farther away from the target will have a lower hit rate due to signal attenuation as defined by Equation 3. Each point on the ROC curve that corresponds to one particular system threshold for making a global decision is defined by a pair of hit rate and false alarm rate measured in one simulation with one million random detection samples. For one particular deployment radius, each sensor is randomly deployed around the target. Sensor distances to the target will not change for the one million samples until next deployment radius. We have the following simulation steps: (i) Under H_1, a random number ranging from 0 to 1 is generated and compared with each individual sensor's hit rate to determine its local binary decision. (ii) If the total number of sensors reporting 1 reaches the system threshold, a target is reported to be present. (iii) The proportion of correct detections out of one million

Table I. Fusion System Performance with Different Deployment Radiuses

$N=25$	$W_{p_{h/f}}$	$P^{8}_{h/f}$	$P^{9}_{h/f}$	$P^{10}_{h/f}$	$P^{11}_{h/f}$	$P^{12}_{h/f}$	$P^{13}_{h/f}$	$P^{14}_{h/f}$	$P^{15}_{h/f}$
R = 1	.64/.2	.99/.11	.99/.05	**.99/.02**	**.98/.01**	.96/.0	.9/.0	.81/.0	.68/.0
R = 3	.51/.2	.96/.11	**.90/.05**	.81/.02	.68/.01	.52/.0	.35/.0	.21/.0	.11/.0
R = 5	.49/.2	**.89/.11**	**.79/.05**	.65/.02	.47/.01	.31/.0	.18/.0	.09/.0	.04/.0

samples is used to approximate the system hit rate. (iv) A similar procedure is performed under H_0 for measuring system false alarm rate. Performance comparisons in terms of two system rates under different system thresholds and deployment radiuses are tabulated in Table I, where the weighted average hit and false alarm rates are denoted as $W_{p_{h/f}}$, and the system hit and false alarm rates with threshold k are represented by $P^{k}_{h/f}$. The calculated threshold bounds are indicated in bold with superior system performance over the weighted averages.

5. CONCLUSIONS

We proposed a threshold fusion method for sensor networks wherein each node decides the presence of a target and sends its binary decision to the fusion center for final decision making. Our method is a centralized hard fusion scheme accepting discrete sensor decisions without requiring a priori probability of target presence. In addition to achieving better system performance than corresponding weighted averages, the determined threshold bounds allow users certain freedom in fine tuning between sensitivity and specificity. The ROC curve from off-line simulations or experiments can be used to maximize the system hit rate under the constraint of a given system false alarm rate or vice versa, and hence to guide practical implementation of sensor networks. Reasonable threshold bounds can be quickly computed at the fusion center without exploring the entire ROC curve and its low computational cost makes practical deployment feasible with limited computing resources.

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Received February 2007; revised August 2007, February 2008, June 2008, January 2009, February 2009; accepted August 2009